WEEKLY TEST TARGET - JEE - TEST - 16
SOLUTION Date 25-08-2019

[PHYSICS]

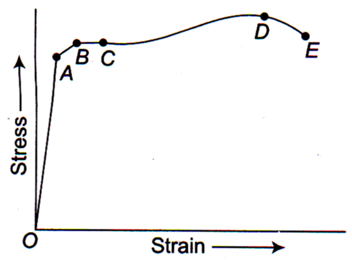
1. As stress is shown on x-axis and strain on y-axis

So we can say that $Y = \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\text{slope}}$

So elasticity of wire P is minimum and of wire R is maximum.

2. In the region OA, the graph is linear showing that stress is proportional to the strain. Is proportional to the strain. Thus, in this region Hooke's law is obeyed.

The point D on the graph is known as ultimate tensile strength.



The point E on the graph is known as fracture point.

3. $Y = \tan\theta$. According to figure $\theta_A > \theta_B > \theta_C$

i.e., $\tan\theta_A > \tan\theta_B > \tan\theta_C$

or $Y_A > Y_B > Y_C$

$\therefore$ A, B, and C graph are for steel, brass and rubber respectively.

4. For a perfectly rigid body, both Young's modulus and bulk modulus is infinite.

5. From the given graph for a stress of 150×10^6 N m^{-2} the strain is 0.002.

$\therefore$ Young's modulus $Y = \frac{\text{Stress}}{\text{Strain}}$

$$Y = \frac{150 \times 10^6}{0.002}\,\text{N m}^{-2} = 7.5 \times 10^{10}\,\text{N m}^{-2}$$

6. Initial length (circumference) of the ring = $2\pi r$
Final length (circumference) of the ring = $2\pi R$
Change in length = $2\pi R - 2\pi r$.

$$\text{Strain} = \frac{\text{change in length}}{\text{original length}} = \frac{2\pi(R-r)}{2\pi r} = \frac{R-r}{r}$$

Now Young's modulus $E = \frac{F/A}{l/L} = \frac{F/A}{(R-r)/r}$

$$\therefore \quad F = AE\left(\frac{R-r}{r}\right)$$

7. Tension in both string shall be same which can be observed by making FBD of string in figure (1)

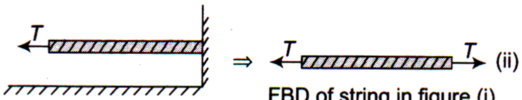


figure (i)
FBD of string in figure (i)

8. $Y = \frac{FL}{\pi r^2 l}$

$\therefore \quad l = \frac{FL}{\pi r^2 Y} \Rightarrow l \propto \frac{L}{r^2}$

$\therefore \frac{L}{r^2}$ is greatest for option A.

9. Shearing strain $= \frac{\Delta x}{L}$

10. If coefficient of volume expansion is α and rise in temperature is $\Delta\theta$ then $\Delta V = V\alpha\Delta\theta \Rightarrow \frac{\Delta V}{V} = \alpha\Delta\theta$

Volume elasticity $\beta = \frac{P}{\Delta V/V} = \frac{P}{\alpha\Delta\theta} \Rightarrow \Delta\theta = \frac{P}{\alpha\beta}$

11 If side of the cube is L then $V = L^3 \Rightarrow \frac{dV}{V} = 3\frac{dL}{L}$

$\therefore$ % change in volume = 3 × (% change in length)

$= 3 \times 1\% = 3\%$ $\therefore$ Bulk strain $\frac{\Delta V}{V} = 0.03$

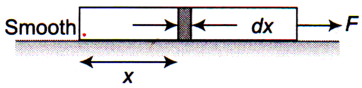
12. At extension l_1, the stored energy $= \frac{1}{2}Kl_1^2$

At extension l_2, the stored energy $= \frac{1}{2}Kl_2^2$

Work done in increasing its extension from l_1 to l_2

$$= \frac{1}{2}K(l_2^2 - l_1^2)$$

13. Tension, $T = \frac{F}{L_0} \cdot x\, x$



Stress, $\sigma = \frac{T}{A} = \frac{F}{AL_0} x$

$$dU = \frac{1}{2} \cdot \frac{\sigma^2}{Y} Adx = \frac{1}{2} \frac{F^2}{A^2 L_0^2} \cdot x^2 \frac{A}{Y} dx$$

or $$dU = \frac{F^2}{2A^2 L_0^2 Y} \cdot x^2 dx$$

$\Rightarrow$ $$U = \frac{F^2}{2AY L_0^2} \int_0^{L_0} x^2 dx$$

$$U = \frac{F^2}{2AY L_0^2} \cdot \frac{L_0^3}{3} = \frac{F^2 L_0}{6AY}$$

14. The elastic potential energy per unit volume

$$= \frac{1}{2} \text{stress} \times \text{strain} = \frac{1}{2} Y \text{ strain} \times \text{strain}$$

$$= \frac{1}{2} Y (\text{strain})^2 = \frac{1}{2} Y \sigma^2$$

15. The energy stored per unit volume is

$$U = \frac{1}{2} \text{stress} \times \text{strain}$$

$$= \frac{1}{2} \text{stress} \times \frac{\text{strain}}{Y}$$

$$U = \frac{(\text{stress})^2}{2Y} = \frac{P^2}{Y}$$

So the correct choice is (b).

16. As the weight of wire acts at centre of gravity.

$\therefore$ Only half the length of wire gets extended.

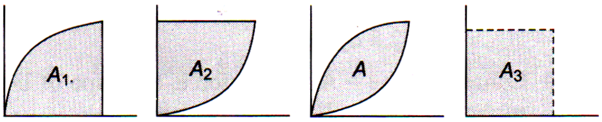
Now $$Y = \frac{F}{A} \cdot \frac{(L/2)}{\Delta l} = \frac{Mg(L/2)}{A \Delta l}$$

$\Rightarrow$ $$\Delta l \frac{MgL}{2AY} \Rightarrow \Delta l \frac{AL\rho g L}{2AY}$$

$\therefore$ $$\Delta l = \frac{PL^2 g}{2Y}$$

So the correct choice is (b)

17. Hysteresis loss corresponding to elasticity per unit volume of a substance is given by the area of hysteresis loop, i.e., stress-strain curve corresponding to one complete loading and deloading.



Area of an ellipse = $\pi\times$ semi-major axis $\times$ semi-minor axis

$$A_1=\frac{1}{4}(\pi\times8\times4\times10^2)\text{ and }A_2=\frac{1}{4}(\pi\times8\times4\times10^2)$$

Also, $A_3 = 8\times4\times10^2$

Area of hysteresis loop is $A = A_1 + A_2 - A_3$

$$A=2\left[\frac{\pi}{4}\times8\times4\times10^2\right]-[8\times4\times10^2]$$

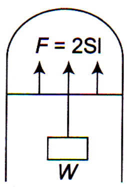
= work done per cycle

= energy lost per cycle per unit volume

18. Because film tries to cover minimum surface area.

19. Here, $W = 1.5\times10^{-2}$ N,

$l = 30$ cm $= 30\times10^{-2}$ m



A liquid film has two free surfaces. A slider will support the weight when the force of surface tension action upwards on the slider ($2Sl$) balances the downward force due to weight (= W)

20. Energy needed = Increment in surface energy

= (surface energy of n small drops) – (surface energy of one big drop)

$= n4\pi r^2T - 4\pi R^2T = 4\pi T(nr^2 - R^2)$

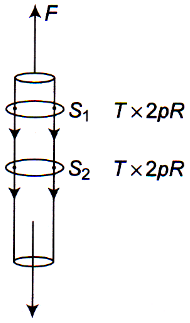
21. $$W=8\pi T(r_2^2-r_1^2)=8\pi T\left[\left(\frac{2}{\sqrt{\pi}}\right)^2-\left(\frac{1}{\sqrt{\pi}}\right)^2\right]$$

$\therefore\quad W=8\times\pi\times30\times\frac{3}{\pi}=720$ erg

22. The force exerted by film on wire or thread depends only on the nature of material of the film and not on its surface area. Hence the radius of circle formed by elastic thread does not change.

23. The free body diagram of the capillary tube is as shown in the figure. Net force F required to hold tube is

F = force due to surface tension at cross-section



Free body daigram of capillary tube

$(S_1 + S_2)$ + weight of tube.

$= (2\pi RT + 2\pi RT) + mg = 4\pi RT + mg$

24. The thin ring is in contact with water from both inside and outside. So, contact length is $2 \times 20 = 40$ cm

$F_{min} = F_{ST} + W = (75 \times 10^{-5}) \times 40 + 0.1 = 0.130$ N

25. It may be noted that the soap film has two free surfaces. So, the effective length is 8ℓ.

26. Effective area $= 2 \times 0.02 \text{ m}^2 = 0.04 \text{ m}^2$

Surface energy $= 5 \text{ m}^{-1} \times 0.04 \text{ m}^2 = 2 \times 10^{-1}$ J

27. $W = [2 \times 4\pi(3r)^2 - 2 \times 4\pi r^2]\, T = 64\, \pi r^2 T$

28. $F = 2\pi r_1 T + F = 2\pi r_2 T$

$= 2\pi(r_1 + r_2)T$

$= 2 \times 3.14(10 + 5)(72) = 6782.4$ dyne

29. $h = \dfrac{2\sigma \cos\theta}{r\rho g}$ or $r = \dfrac{2\sigma \cos\theta}{h\rho g}$

or $r = \dfrac{2 \times 75 \times 10^{-3} \times \cos 0°}{3 \times 10^{-2} \times 10^3 \times 10} \text{m} = 5 \times 10^{-4}$ m

30. $h = h_0 = \dfrac{2T \cos\theta}{\rho g r}$

$= \dfrac{2(72)\cos 0°}{(1)(1000)\left(\dfrac{1}{40}\right)} = 57.6$ cm

Since ℓ (= 50 cm) < h_0.

$h = 50$ cm

[CHEMISTRY]

54.

Volume of O_2 diffused $= \frac{22400 \times 0.48}{32} = 336$ mL.

Let the volume of CO_2 diffused be x mL.

Rate of diffusion of $O_2 = \frac{336}{1200}$ mL s^{-1}.

Rate of diffusion of $CO_2 = \frac{x}{1200}$ mL s^{-1}.

$$\frac{r_{O_2}}{r_{CO_2}} = \frac{V_{O_2}/t}{V_{CO_2}/t} = \sqrt{\frac{M_{CO_2}}{M_{O_2}}}$$

or $$\frac{\frac{336}{1200}}{\frac{x}{1200}} = \sqrt{\frac{44}{32}}.$$

$\therefore \; x = 286.5$ mL

55.

$(U_{rms})_1 = \sqrt{\frac{3RT_1}{M_1}}$ for N_2 molecule, mol. wt. $M_1 = 28$.

$(U_{rms})_2 = \sqrt{\frac{3RT_2}{M_2}}$ for N atom, $M_2 = 14$.

$$\frac{(U_{rms})_1}{(U_{rms})_2} = \frac{\sqrt{\frac{3RT_1}{M_1}}}{\sqrt{\frac{3RT_2}{M_2}}} = \sqrt{\frac{3RT_1}{M_1} \times \frac{M_2}{3RT_2}} = \sqrt{\frac{T_2 \times 14}{28 \times 2T_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}.$$

$(U_{rms})_2 = 2(U_{rms})$.

56.

The van der Waals equation for n moles of a real gas is given by

$$\left(p + \frac{n^2 a}{V^2}\right)(V - nb) = nRT \quad \text{or} \quad \left(p + \frac{a}{V_m^2}\right)(V_m - b) = RT,$$

where V_m = molar volume = V/n.

At low pressure, V_m is high and so b can be neglected.

The $\left(p + \frac{a}{V_n^2}\right)V_m = RT \qquad$ or $\qquad pV_m = \frac{a}{V_m} = RT$

$\Rightarrow pV_m = RT - \frac{a}{V_m} \qquad \Rightarrow \frac{pV_m}{RT} = Z = 1 - \frac{a}{RTV_m}.$

$Z = 1 - \frac{ab}{RT} \quad \left(\because \; V \propto \frac{1}{p}\right).$

57.

For n moles of a real gas, the van der Waals equation becomes

$$\left(p+\frac{a}{V_m^2}\right)(V_m-b)=RT$$

At high temperature and low pressure, V_m is large in comparison to b and $\frac{a}{V_m^2}$ is negligibly small in comparison to p. Hence the above equation is reduced to $pV_m = RT$.

58.
59.
60.

The less the value of a, the weaker is the intermolecular attraction.

[MATHEMATICS]

61. Since, $[x^3]$ is not continuous and differentiable at integral point. So, $f(x)$ is continuous and differentiable in $[4, 6]$ if $\left[\frac{(x-2)^3}{a}\right]=0 \Rightarrow a \geq 64$.

62. $\frac{\cos x}{|\cos x|}$ is not defined at $x=(2n+1)\frac{\pi}{2}, \forall n \in I$. Hence, discontinuous.

63.

$f(0)=0$ and $f(x)=xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}$

R.H.L. $= \lim_{h\to 0}(0+h)e^{-2/h} = \lim_{h\to 0}\frac{h}{e^{2/h}}=0$

L.H.L. $= \lim_{h\to 0}(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}=0$; $\therefore f(x)$ is continuous.

$$Rf'(x)=\lim_{h\to 0}\frac{(0+h)e^{-\left(\frac{1}{h}+\frac{1}{h}\right)}-he^{-\left(\frac{1}{h}+\frac{1}{h}\right)}}{h}=0$$

$$Lf'(x)=\lim_{h\to 0}\frac{(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}-he^{-\left(\frac{1}{h}+\frac{1}{h}\right)}}{-h}=1$$

$\Rightarrow Lf'(x)\neq Rf'(x)$. $f(x)$ is not differentiable at $x=0$.

64.

$$\lim_{x\to\frac{\pi}{4}} f(x)=\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{4x-\pi}, \left[\frac{0}{0}\text{ form}\right]$$

$$\lim_{x\to\frac{\pi}{4}}\frac{-\sec^2 x}{4}=\frac{-2}{4}=\frac{-1}{2}.$$

$\therefore$ For $f(x)$ to be continuous at $x=\frac{\pi}{4}, f\left(\frac{\pi}{4}\right)=\frac{-1}{2}$

65. $\frac{x}{1+|x|}$ is always differentiable

$\Rightarrow (x-2)(x+4)|(x-1)(x-2)(x-3)|$ is not differentiable at $x=1, 3$. So, $f(x)$ is not differentiable at $x=1, 3$.

66. $\lim_{x\to\frac{\pi}{2}-} f(x) = \lim_{h\to 0}\left(\frac{\sin(\sin h)}{-h}\right) = -1$

$\Rightarrow \quad \lim_{x\to\frac{\pi}{2}+} f(x) = \lim_{h\to 0}\frac{\sin(-\sin h)}{h} \neq -1$

So, $\lim_{x\to\frac{\pi}{2}} f(x)$ does not exist.

67. $\lim_{x\to 0-} f(x) = 1+1 = 2, \ \lim_{x\to 0+} f(x) = 0, \ f(0) = 2$

68. $\lim_{x\to 1-} f(x) = a-b, \ \lim_{x\to 1+} f(x) = 2 \Rightarrow a-b = 2$

All the given sets of a, b make $f(x)$ continuous at $x=1$.

69. $f(g(x)) = \frac{1}{\left(\frac{1}{x^2}-1\right)\left(\frac{1}{x^2}-2\right)} = \frac{x^4}{(1-x^2)(1-2x^2)}$

$\Rightarrow f(g(x))$ is discontinuous at $x=\pm 1, x=\pm\frac{1}{\sqrt{2}}$ and $x=0$

Since, $g(x)$ is discontinuous at $x=0$.

70.
$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{f(x)+f(h)-f(x)}{h}$$
$$(\because f(x+y) = f(x)+f(y))$$
$$= \lim_{h\to 0}\frac{f(h)}{h} = \lim_{h\to 0}\frac{h^2 g(h)}{h} = 0 \qquad (\because f(x) = x^2 g(x))$$

71. $\lim_{x\to 2-} f(x) = 3, \ \lim_{x\to 2+} f(x) = 3$ and $f(2) = 3$.

72. Here $f\left(\frac{3\pi}{4}\right) = 1$ and $\lim_{x\to 3\pi/4-} f(x) = 1$

$$\lim_{x\to 3\pi/4+} f(x) = \lim_{h\to 0} 2\sin\frac{2}{9}\left(\frac{3\pi}{4}+h\right) = 2\sin\frac{\pi}{6} = 1$$

Hence $f(x)$ is continuous at $x = \frac{3\pi}{4}$.

73.
$$\because f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{f(x)f(h)-f(x)}{h}$$
$$= f(x)\lim_{h\to 0}\frac{f(h)-1}{h}$$
$$= f(x)f'(0)$$
$$f'(3) = f(3)f'(0)$$
$$= 3\times 11 = 33$$

74. $\lim_{x\to 1}\frac{\{t^2\}_4^{f(x)}}{(x-1)} = \lim_{x\to 1}\frac{\{f(x)\}^2-16}{(x-1)}$

$= \lim_{x\to 1}\frac{2f(x)f'(x)-0}{1} = 2f(1)f'(1)$

$= 2\times 4\times f'(1)$

$= 8f'(1)$.

75.

$$f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0\end{cases},\quad g(x)=\begin{cases} x^2\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0\end{cases}$$

$L\,f'(0)=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$

$=\lim_{h\to 0}\frac{(0-h)\sin(-\frac{1}{h})-(0)}{-h}=\lim_{h\to 0}-\sin\left(\frac{1}{h}\right)$

= a quantity which lies between – 1 and 1

$R\,f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$=\lim_{h\to 0}\frac{(0+h)\sin\frac{1}{h}-0}{h}=\lim_{h\to 0}\sin\frac{1}{h}$

= a quantity which lies between – 1 and 1

Hence $L\,f'(0)\neq R\,f'(0)$

$\therefore f(x)$ is not differentiable at $x=0$

Now $L\,g'(0)=\lim_{h\to 0}\frac{f(0-h)-f(0)}{0-h}$

$L\,g'(0)=\lim_{h\to 0}\frac{(0-h)^2\sin(-\frac{1}{h})-0}{-h}=\lim_{h\to 0}h\sin\left(\frac{1}{h}\right)$

$L\,g'(0)=0\times\left(-1\leq\sin\frac{1}{h}\leq 1\right)\Rightarrow L\,g'(0)=0$

and $Rg'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{(0+h)^2\sin\left(\frac{1}{h}\right)-0}{h}$

$=\lim_{h\to 0}h\sin\left(\frac{1}{h}\right)=0\times\left(-1\leq\sin\left(\frac{1}{h}\right)\leq 1\right)=0$

$\because L\,g'(0) = R\,g'(0)$ then $g(x)$ is differentiable at $x = 0$

Now $\qquad g(x) = x^2 \sin \frac{1}{x}$

$g'(x) = 2x \sin \frac{1}{x} + x^2 \cos \frac{1}{x} \times -\frac{1}{x^2}$

$g'(x) = 2x \sin \frac{1}{x} - \cos \frac{1}{x} \Rightarrow g'(x) = 2f(x) - \cos \frac{1}{x}$

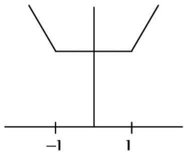
So, $g'(x)$ is not differentiable at $x = 0$.

76. $f(x) = \max \{(1-x), (1+x), 2\}, \forall\, x \in (-\infty, \infty)$.

$$f(x) = \begin{cases} 1+x; & x > 1 \\ 2; & -1 \le x \le 1 \\ 1-x; & x < -1 \end{cases}$$

Since $f(x) = 1-x$ or $1+x$ are polynomial functions and $f(x) = 2$ is a constant function.

$\therefore$ These are continuous at all points .....(i)



$\therefore f(x)$ is differentiable at all the points, except at $x = 1$ and at $x = -1$ .....(ii)

77. Given, $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$

Replacing x by $3x$ and y by zero,

then $\qquad f(x) = \frac{f(3x)+f(0)}{3}$

$\Rightarrow \qquad f(3x) - 3f(x) = -f(0)$

and $\qquad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{f\left(\frac{3x+3h}{3}\right) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{f(3x) + f(3h)}{3} - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(3x) + f(3h) - 3f(x)}{3h}$$

$$= \lim_{h \to 0} \frac{f(3h) - f(0)}{3h} \quad \text{[from Eq. (i)]}$$

$$= f'(0)$$

$$= 3$$

$\therefore \quad f(x) = 3x + c$

$\Rightarrow \quad f(0) = 0 + c = 3$

$\therefore \quad c = 3$

Then, $f(x) = 3x + 3$

Hence, $f(x)$ is continuous and differentiable everywhere.

78. $\because [(2x - 3\pi)^3] =$ Integer (say n)

$\therefore \quad \tan n\pi = 0$

and $\quad 1 + [2x - 3\pi]^2 \neq 0$

Hence, $\quad f(x) = 0$

Hence, $f(x)$ is continuous and differentiable for all $x \in R$.

79. $\lim_{x \to \pi/2-} f(x) = \frac{\pi}{2}, \lim_{x \to \pi/2+} f(x) = \frac{-\pi}{2}$ and $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$.

80.

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0-} \left(\frac{2 \sin^2 2x}{(2x)^2} \right) 4 = 8$$

$\lim_{x \to 0+} f(x) = \lim_{x \to 0+} \sqrt{16 + \sqrt{x}} + 4 = 8$. Hence $a = 8$.

$\lim_{x \to 1-} f(x) = a - b, \lim_{x \to 1+} f(x) = 2 \Rightarrow a - b = 2$

81. $\text{RHL} = \lim_{x \to 0^+} f(x)$

$$= \lim_{h \to 0} f(0 + h)$$

$$= \lim_{h \to 0} h^n \sin\left(\frac{1}{h}\right)$$

$= 0^n \sin(\infty)$

$= 0^n \{-1 \text{ to } 1\}$

$\because$ V. F. $= f(0) = 0$

$\therefore$ $n > 0$ ..(i)

$$Rf'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0} \frac{h^n \sin\left(\frac{1}{h}\right) - 0}{h}$$

$$= \lim_{h\to 0} h^{n-1} \sin\left(\frac{1}{h}\right)$$

$= 0^{n-1} \sin\infty$

$= 0^{n-1} (-1 \text{ to } 1)$

For not differentiable

$n - 1 \le 0$

$\therefore$ $n \le 1$

From Eqs. (i) and (ii),

$0 < n \le 1$

$\therefore$ $n \in (0, 1]$

82. We have,

$$f(x) = \begin{cases} [\cos \pi x], & x < 1 \\ |x-2|, & 1 \le x < 2 \end{cases}$$

$= 2 - x, 1 \le x < 2$

$$\begin{cases} -1, & \frac{1}{2} < x < 1 \\ 0, & 0 < x \le \frac{1}{2} \\ 1, & x = 0 \\ 0, & -\frac{1}{2} \le x < 0 \\ -1, & -\frac{3}{2} < x < -\frac{1}{2} \end{cases}$$

It is evident from the definition that $f(x)$ is discontinuous at $x = 1/2$.

83.

We have, $f(x) = |x| + |x-1|$

$$= \begin{cases} -2x+1, & x < 0 \\ x - x + 1, & 0 \le x < 1 \\ x + x - 1, & x \ge 1 \end{cases} = \begin{cases} -2x+1, & x < 0 \\ 1 & 0 \le x < 1 \\ 2x-1, & x \ge 1 \end{cases}$$

Clearly, $\lim_{x\to 0^-} f(x) = 1$, $\lim_{x\to 0^+} f(x) = 1$, $\lim_{x\to 1^-} f(x) = 1$

and $\lim_{x\to 1^+} f(x) = 1$. So, $f(x)$ is continuous at $x = 0, 1$.

$$\text{Now } f'(x) = \begin{cases} -2, & x < 0 \\ 0, & 0 \le x < 1 \\ 2, & x \ge 1 \end{cases}$$

Here $x = 0$, $f'(0^+) = 0$ while $f'(0^-) = -2$

and at $x = 1$, $f'(1^+) = 2$ while $f'(1^-) = 0$

Thus, $f(x)$ is not differentiable at $x = 0$ and 1.

84. $\lim_{x\to a-} f(x) = -1,\ \lim_{x\to a+} f(x) = 1,\ f(a) = 1.$

85. Let $x^3 = n \in I$

$\therefore$ $x = n^{1/3}$

then, $f(x) = (-1)^n = \pm 1$

86. $\because 0 < x \le \dfrac{\pi}{2}$

$\therefore 0 \le \cos x < 1$

then $[\cos x] = 0$

$\therefore f(x) = 1$

Hence, $f(x)$ is continuous in $(0, \pi/2)$.

87. $\lim_{x\to 1} f(x) = 1,\ f(1) = 2.$

88. $\because\ y^2 - 4y + 11 = (y-2)^2 + 7$

$\therefore$ $\min(y^2 - 4y + 11) = 7$ (at $y = 2$)

$\therefore$ $\lim_{x\to 0}\left[\min(y^2 - 4y + 11)\dfrac{\sin x}{x}\right]$

$= \lim_{x\to 0}\left[\dfrac{7\sin x}{x}\right]$

Now, let $f(x) = \left[\dfrac{7\sin x}{x}\right]$

For $x > 0$

$\sin x < x$

$\therefore$ $\dfrac{\sin x}{x} < 1$

$\Rightarrow$ $\dfrac{7\sin x}{x} < 7$

$\Rightarrow$ $\left[\dfrac{7\sin x}{x}\right] = 6$

$\therefore$ RHL $= \lim_{x\to 0+} f(x) = 6$

For $x < 0$

$\sin x > x$

$\dfrac{\sin x}{x} < 1$

$\therefore$ $\dfrac{7\sin x}{x} < 7$

$\Rightarrow$ $\left[\dfrac{7\sin x}{x}\right] = 6$

$\therefore$ LHL $= \lim_{x\to 0-} f(x) = 6$

Hence,

$$\lim_{x\to 0}\left[\min(y^2 - 4y + 11)\frac{\sin x}{x}\right] = 6$$

89. $\lim_{x \to \pi/2} \frac{\sin(x\cos x)}{\sin\left(\frac{\pi}{2} - x\sin x\right)}$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sin(x\cos x)}{(x\cos x)} \cdot \frac{x\cos x}{\sin\left(\frac{\pi}{2} - x\sin x\right)} \cdot \frac{\left(\frac{\pi}{2} - x\sin x\right)}{\left(\frac{\pi}{2} - x\sin x\right)}$$

$$= 1 \cdot 1 \cdot \lim_{x \to \pi/2} \frac{x\cos x}{\left(\frac{\pi}{2} - x\sin x\right)}$$

Put $x = \pi/2 + h$

then $= \lim_{h \to 0} \frac{\left(\frac{\pi}{2} + h\right)\cos\left(\frac{\pi}{2} + h\right)}{\frac{\pi}{2} - \left(\frac{\pi}{2} + h\right)\sin\left(\frac{\pi}{2} + h\right)}$

$$= \lim_{h \to 0} \frac{-\left(\frac{\pi}{2} + h\right)\sin h}{\frac{\pi}{2}(1 - \cos h) - h\cos h}$$

$$= \lim_{h \to 0} \frac{-\left(\frac{\pi}{2} + h\right)\left(\frac{\sin h}{h}\right)}{\frac{\pi}{2}\frac{(1 - \cos h)}{h} - \cos h}$$

(divide above and below by h)

$$= \frac{-\left(\frac{\pi}{2} + 0\right) \cdot 1}{0 - 1} = \frac{\pi}{2}$$

90. Divide above and below by x^m, then

$$k = \lim_{x \to \infty} \left(\frac{\sum_{k=1}^{1000} \left(1 + \frac{k}{x}\right)^m}{1 + \frac{10^{1000}}{x^m}} \right)$$

$$= \frac{1 + 1 + 1 + \ldots \text{ upto 1000 times}}{1 + 0}$$

$$= 1000 = 10^3$$